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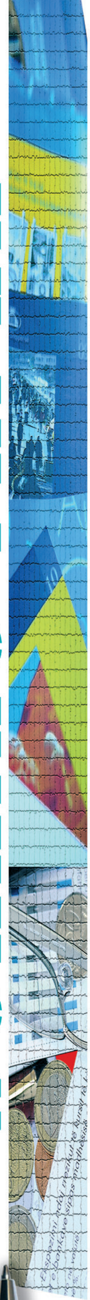

AN ESTIMATION OF THE NATURAL INTEREST RATE IN ALBANIA

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ABSTRACT

We estimate the natural rate of interest in Albania using the Bayesian Maximum Likelihood method and the Kalman Filter. The structure of the model used to identify the behaviour of the natural interest rate in time is based on the seminal work of Laubach and Williams (2003) and includes a joint estimation of the output gap and of potential growth. The potential growth rate is the primary structural driver of the natural interest rate level in time, whereas the output gap defines its volatility around this level through evolving preferences on interest rates. Empirical results indicate a drop in the natural rate of interest in Albania after the economic and financial crisis linked to the deceleration in potential growth and to worsening cyclical conditions.

1 – INTRODUCTION

In 1898, Swedish economist Knut Wicksell, first coined the term "natural rate of interest" as the interest rate at which *"There is a certain rate of interest on loans which is neutral with respect to commodity prices, and tends neither to raise nor to lower them."* (Wicksell (1898)). By having no inflationary or deflationary pressures on the price levels, Wicksell believed that the natural rate of interest is determined by capital investment rates in the real sector of the economy *"The rate of interest at which the demand for loan capital and the supply of savings exactly agree, and which more or less corresponds to the expected yields on the newly created real capital, will then be the normal or natural rate."* (Wicksell (1934), pg193). If loan rates diverge from the natural rate (e.g. expansionary bank lending of "fictitious" deposits), a cumulative process of price adjustments from the real sector of the economy will ensue. Market agents will borrow at the loan interest rate and invest in capital which yields the natural rate, producing an increase in output and prices until the two interest rates converge.

In contrast to the so called Wicksellian natural rate of interest, Keynes took his turn in identifying a "neutral" or "optimum" rate of interest *"which prevails in equilibrium where output and employment are such that the elasticity of employment as a whole is zero"* (Keynes (1936)). In other words, the author refers to the modern day view of the economy operating at potential levels and at full employment. This is in contrast to Wicksell's view of the natural rate of interest that brings stable prices but not full employment.

Moreover, the natural rate of interest could be viewed as a short term or medium term phenomenon. A short-term natural rate of interest represents the level of interest rates at which inflation is stable each period and such rates can be influenced by temporary shocks to the supply and demand dynamics of the economy. Instead, the medium term concept relates more to the natural rate of interest that is determined only by structural factors of the real economy. As short term, temporary, shocks to the real economy fade away in the medium term, such natural interest rate is affected primarily by demographic factors and productivity in the medium term.

For the purpose of this study, and in line with latest monetary policymaking models, we identify the natural rate of interest as a medium term interest rate at which the economy operates at potential levels, inflation reaches its target and all transitory shocks to the real economy have dissipated. The identification of the medium-term natural interest rate will represent both the "natural" and the "neutral" rate discussed above. This rate of interest, provides no inflationary or deflationary pressure on the price level, and will give way to a more efficient interpretation of the monetary policy stance. If the actual short term interest rate stands below (above) the natural interest rate, then monetary policy is deemed to be expansionary (contractionary).

In recent years, past the latest global financial crisis, a large number of both developed and developing countries, Albania included, have operated in an environment characterized by very low (sometimes negative) rates of interest. In addition, many global economies both developed and developing, have observed secular declining trends in such rates since the early 90's (Carvalho et al. (2016)). Laubach and Williams (2003)[1], published a seminal research paper that identified empirically the factors behind the secular decline in natural rates of interest. Furthermore, they introduced a methodological framework easily replicable for countries at different stages of development and with diverse economic structures.

In Albania, the empirical investigation of the evolution of the natural rate of interest has not gained enough attention. Research has mainly focused on the evolution of potential output, which is nevertheless an important structural determinant of the natural rate of interest. Trend growth rates have exhibited a secular decline during the past decade and a half (Çeliku et al. (2018)). As such, it becomes ever more important for policymaking to empirically pin down the behaviour of the natural interest rate in Albania.

[1] *Referred to as LW from this point on.*

In this paper, we investigate the evolution of the natural rate of interest in Albania for the period 2001 to 2017. We adopt the methodological framework introduced by LW (2003) and augment it with the real exchange rate, in order to capture the overall Albanian economic structure as a small open economy and the importance it plays in business cycle fluctuations and inflation dynamics. We initially use a three-step process to test empirically for structural breaks in the unobserved series of natural rate of interest and of trend GDP growth rate and, we use this information to impose restrictions on the signal-to-noise ratios during the estimation phase. For that, we follow a Bayesian Maximum Likelihood (ML) strategy to estimate model parameters and thereafter run a two-sided Kalman Filter (KF) in order to estimate the evolution of the natural interest rate alongside the other unobserved variables.

Our results show that the natural interest rate has declined starting from 2007, primarily linked to a deceleration of trend growth but also due to a downward shift of preferences on interest rates. The natural interest rate is estimated to have reached its minimum in 2014, in line with the trough in the Albanian business cycle, and to have started a slow pickup thereafter, mirroring the estimation of a slowly accelerating trend GDP growth rate. We estimate the pre-crisis natural rate of interest to have fluctuated around an average of 2.8% per year and for it to have dropped on average to about 1.2% per year thereafter. The empirical framework introduced here provides a new additional framework for policymakers in Albania to be able to track, with some degree of certainty, the behaviour of the natural rate of interest in time and the implicit evolution of the neutral policy rate. This knowledge set complements the current methodologies in place, in helping in the measurement of the intensity of the current monetary policy impulse and of the future proposed changes with respect to the inflation target.

The paper is structured as follows. Section 2 reviews the literature on the estimation of the natural rate of interest. Next, in Section 3, we elaborate on the model used and its equation structure. Section 4 introduces the estimation methodology and section 5 the results. Section 6 provides a short deliberation on monetary policy implications of the natural rate of interests and we conclude in section 7.

2 – LITERATURE REVIEW

The initial publishing of the Laubach and Williams (2003) study paved the way for a plethora of research about the natural interest rate developments and its linkage with the estimation of potential output. The LW (2003) results inspired many authors internationally to investigate further and shed light on the aspects that determine the natural rate of interests, both in developed and emerging economies. The majority of such research identified declining natural interest rates for the past two decades in advanced economies due to aging populations, higher demand for savings and declining potential growth, such as in Manrique and Marquez (2004), Benati and Vitale (2007), Holston et al (2016) and Zhu et al (2016). Similar empirical results were obtained from research on the Central Eastern European Countries and other countries in the region. Grafe et al (2018) concluded that natural interest rates have been declining in CEEMEA[2] countries since the '90s. Several country-specific studies provide the same results such as Hledik and Vlcek (2017) for the Czech Republic, Us (2018) for Turkey, Acatrinei et al (2018) for Romania, Baksa et al (2013) for Hungary, Grui et al (2018) for Ukraine and Stafanski (2017) for the CEE region.

As we identify the natural rate of interest as the interest rate that brings no inflationary or deflationary pressure on prices, and output is at its equilibrium level, there exist different methods to estimate such unobserved variable. The most trivial approach would be to estimate the natural rate of interest using simple moving averages. Laubach and Williams (2003) stipulate that the former method (long-term average of actual real interest rates) is suitable if the natural interest rate is somewhat constant over the past and there are no clear trends in inflation developments. Nevertheless, if supply and demand dynamics change in a large and persistent way, then the simple average over the sample period might be deemed not efficient in estimating the natural rate of interest. The latter method, applying historical moving averages, allows for persistent changes in the natural interest rate dynamics. Nevertheless, Laubach and

[2] *CEEMEA countries in this research consist of: Israel, Poland, Czech Republic, Hungary, Turkey, Romania, Russia and South Africa*

Williams (2015) showed that, by using the 10 year moving average of ex-post real short-term interest rates and the median real short-term interest rates across 17 developed countries, there exists a high degree of persistence of sizable swings in the average real interest rate dynamics.

More complex techniques use univariate time-series filters (HP filter, BP filter etc.) that draw distinctions between short-term fluctuation and long-term movements. Such statistical filtering techniques track long-term dynamics of short-term interest rates. They are efficient mainly in times when supply and demand factors are relatively stable, but do not make up for a natural interest rate free of any inflationary or deflationary pressure when supply and demand dynamics fluctuate. More specifically, these univariate filters would tend to underrate (overrate) the natural rate of interest in times of high (low) inflation if the short-term interest rates are below (above) the natural rate. In addition, these univariate filters would also suffer from end-point bias. Consequently, they would interpret the low interest rate environment present after the latest financial crisis, mainly as a trend development, estimating very low natural interest rate values at the end of the sample (Hamilton et al (2015); Laubach and Williams (2015)).

Another method to provide point estimates on the natural rate of interest is by using forward rates implied from long-term government securities yields. In theory, the latter tend to incorporate views of market agents about the future natural interest rate and diverge from the current values of the short-term interest rate. Nevertheless such implied forward rates take into consideration also a term premium, which tends to impair the estimation process of natural interest rates because the premium fluctuations affect the variations of future forward rates (Kim and Wright (2005)).

Other advanced methods include using structural or semi-structural models to estimate the natural rate of interest by explicitly incorporating movements in inflation, output and interest rates. Such models help provide a full picture of the economy by imposing restrictions and relationships between economic indicators based on economic theory. In between these methods, DSGE models (e.g.

Barsky et al (2014), Del Negro et al (2015), Haavio et al (2016)) and time-varying parameter (TVP) VARs and structural VARs (Lubik and Mathes (2015), Brzoza-Brzezina (2003), Elomari and Van Oost (2018)) have been used in order to estimate of the natural rate of interest.

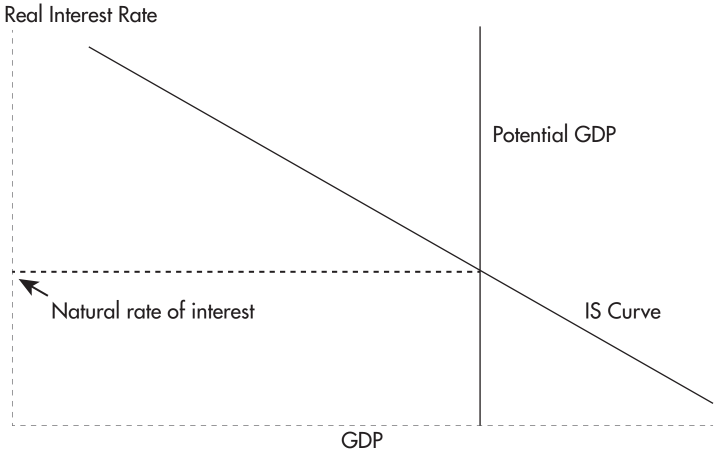
For the purpose of this study, we have chosen to construct a semi-structural model based on the literature that follows on the work of Laubach and Williams (2003) and jointly estimates the natural rate of interest, together with potential output and trend growth, using a Bayesian Maximum Likelihood technique and Kalman Filter. On the one hand, such models diverge from simple statistical methods and univariate filters by allowing the natural rate of interest and other estimates to vary in time and be linked directly to real economy indicators. On the other hand, it requires less theory-based restrictions on the relationships between economic variables than those needed in state-of-the-art DSGE models. Being one of the first methods to be widely used in estimating the natural rate of interest, the Laubach and William (2003) model has been altered, tweaked and updated by other researchers following the same methodological framework. In addition, the model itself has been revisited twice by the authors (Laubach and Williams (2015) and Holston et al (2016)). However, the original model, taken in a-priori form, should be approached with caution as it does not include foreign variables and shows maximum efficiency for closed economies. In trying to tackle with the major shortcoming of using such a model, various adjustments have been done to the original LW (2003) model to bring it closer to the characteristics of the Albanian economy and to incorporate new variables into the mix. Nevertheless, as this study represents the first try at measuring the natural rate of interest in Albania, model specifications and the results should provide some solid ground on which further research could be carried.

3 – THE MODEL

3.1 Model Structure

Laubach and Williams (2003) motivation behind constructing their model stemmed from the substantial increase in the level of potential GDP in the U.S. during the mid-1990s, which lead the authors to question whether this movement represented a level shift or a slope change of potential growth. Different answers would have different implications in policy formulation and implementation. Moreover, the authors faced added uncertainties about the time-varying nature of the so-called r^*, the natural rate of interest. In order to tackle such issues and more, Laubach and Williams (2003) developed a model to jointly estimate three unobserved variables, which are valuable to monetary policymaking; (i) the natural rate of interest (ii) potential output and (iii) trend growth rate by utilizing the relationships between the output gap, inflation and real interest rates. The economic theory behind such model is based upon the neoclassical growth theory, whereas the medium-term natural rate of interest is defined as an increasing function of the trend growth rate of output as depicted in Figure 1.

Figure 1 – Identification of the natural rate of interest



Source: Laubach and Williams (2003)

Real interest rate movements show direct effects on the real economy (consumption and investments), represented by the IS curve in figure 1. As potential output remains constant, the intersection point with the IS curve represent the natural rate of interest. Judging it from a medium-term perspective, the natural rate of interest is allowed to move only due to structural changes in the real economy, and would not be subject to variations ranging from temporary (cyclical) movements in it. In order to draw a difference between the short-term fluctuations and long-term movements, real variables would be based upon a gap form to identify the natural interest rate, such as:

$$x_t\,(actual)=\bar{x}_t\,(trend)+\hat{x}_t\,(gap)$$

Such decomposition allows for real variables to be decomposed into a trend component, driven by fundamental (structural) factors, and a gap component, measured in percentage deviation from equilibrium and representing the cyclical (transitory) movements of the variable. When the actual values of the real variable equal its trend (equilibrium) values, the variable is said to have zero gap. As the trend component is determined by country specific fundamentals, monetary policy has no effect and is not expected to react to fluctuations in such variables, but should react towards movements in the gap variable.

3.2 Model Equations

In order to estimate the natural rate of interest in Albania, an adjusted Laubach and Williams (2003) model is crafted (reduced-form New Keynesian model), comprised of the following equations:

$$\hat{y}_t=\alpha_1 * \hat{y}_{t-1} - \alpha_2 * (rr_{t-1}-r\widetilde{r}_{t-1})+\alpha_3 * \hat{Z}_{t-1}+\varepsilon_t^{\hat{y}} \qquad (1)$$

$$\Delta\pi_t=\beta_1 * \Delta\pi_{t-1}+\beta_2 * \hat{y}_{t-1}+\beta_3 * \hat{Z}_t+\varepsilon_t^{\Delta\pi} \qquad (2)$$

$$r\widetilde{r}_t=\rho * \Delta\bar{y}_t / 4+\theta_t \qquad (3)$$

$$\theta_t= \nu * \theta_{t-1}+\varepsilon_t^{\theta} \qquad (4)$$

$$\bar{y}_t = \bar{y}_{t-1} + \overline{\Delta y}_t/4 + \varepsilon_t^{\bar{y}} \quad (5)$$

$$\Delta\bar{y}_t/4 = \Delta\bar{y}_{t-1}/4 + \varepsilon_t^{\overline{\Delta y}/4} \quad (6)$$

$$y_t = \bar{y}_t + \hat{y}_t \quad (7)$$

$$rr_t = \gamma_1 * rr_{t-1} + \varepsilon_t^{rr} \quad (8)$$

$$\hat{Z}_t = \gamma_2 * \hat{Z}_{t-1} + \varepsilon_t^{\hat{Z}} \quad (9)$$

The main unobserved variables to be jointly estimated are the output gap ($\hat{y}_t$), potential output ($\bar{y}_t$), the trend GDP growth rate ($\overline{\Delta y}_t/4$) and the natural rate of interest ($\widetilde{rr}_t$).

Equation 1 describes the behavioral relationship of the output gap ($\hat{y}_t$), modeling it on its first autoregressive component that captures real rigidities in the economy, the monetary conditions index (lagged real interest rate gap ($rr_{t-1} - \widetilde{rr}_{t-1}$) and real exchange rate gap ($\hat{Z}_{t-1}$)), and an IID exogenous shock ($\varepsilon_t^{\hat{y}}$)[3].

$$\hat{y}_t = \alpha_1 * \hat{y}_{t-1} - \alpha_2 * (rr_{t-1} - \widetilde{rr}_{t-1}) + \alpha_3 * \hat{Z}_{t-1} + \varepsilon_t^{\hat{y}} \quad (1)$$

The output gap, used as a proxy for business cycle fluctuations, takes into consideration the effect of two of the most important monetary policy transmission channels, the interest rate channel and the exchange rate channel. The former channel handles the intertemporal substitution effect of consumption and investment, while the latter deals with the substitution effect between domestic and foreign goods. The real exchange rate is incorporated in this model as an exogenous variable (estimated outside the model). It is deemed to be a worthy inclusion in the overall model, as it provides an added layer of information that fits well in the overall Albanian economic structure as a small open economy. Such modifications to capture the nature of small open economies can also be found in Hledik and Vlcek (2017) and in Grui et al (2018).

[3] *As a general rule throughout this paper, (^) notation represents the variable in gap form, (¯) notation represents the variable in trend form, (~) notation represents the natural rate of interest and (ε_t^x) notation represents an exogenous IID shock to variable x at time t.*

Equation (2) models the fluctuations of inflation rates ($\Delta\pi_t$) on inflation persistence ($\Delta\pi_{t-1}$), the cyclical conditions of the economy, defined by the lagged output gap ($\hat{y}_{t-1}$) representing real marginal costs of domestic producers and the relative price cycles. The latter is defined by the real exchange rate gap ($\hat{Z}_t$) as a proxy for the real marginal costs of importers[4]. An IID exogenous shock ($\varepsilon_t^{\Delta\pi}$) is included in the equation. In more details:

- An increase (decrease) in domestic demand drives firms to utilize more (less) spare capacity, and in turn, further increases (decreases) in demand beyond its potential level, would bring higher (lower) production costs per unit which would make firms increase (decrease) prices in order to accommodate costs.
- A depreciation (appreciation) of the exchange rate opens a positive (negative) REER gap and increases (decreases) the costs of importers. In turn, prices would go up (down) in order for firms to restore profit margins.

$$\Delta\pi_t=\beta_1 * \Delta\pi_{t-1} + \beta_2 * \hat{y}_{t-1} + \beta_3 * \hat{Z}_t + \varepsilon_t^{\Delta\pi} \qquad (2)$$

Equation (3) models the natural rate of interest on the trend growth rate of the economy ($\Delta\bar{y}_t /4$), representing fluctuations in capital, labor and total factor productivity and a preference variable (θ_t) as a proxy for demand and supply of savings.

$$\widetilde{rr}_t = \rho * \Delta\bar{y}_t /4 + \theta_t \qquad (3)$$

The preference variable measures factors mainly related to the time preference of households (Laubach and Williams (2003)). Conceptually, the latter variable captures the balance of demand and supply of savings during the business cycle and the effect it has on the natural rate of interest (Garnier and Willhelmsen (2005)). In periods of cyclical upturn, demand for savings is higher, pushing up preferences on the market-clearing interest rate on the supply side. The opposite holds true in periods of cyclical downturn. The preferences variable is modeled (Eq. 4) on an autoregressive process AR (1) and a serially uncorrelated error (ε_t^{θ}).

[4] *These two cyclical determinants together comprise the so-called Real Marginal Costs (RMC) component.*

Potential output ($\Delta\bar{y}_t$) is represented by a random walk process with a drift (Eq. 5) captured through an AR (1) component, the trend GDP growth rate and an error ($\varepsilon_t^{\hat{y}}$). Trend GDP growth rate (Eq. 6) is specified as a random walk process with a serially uncorrelated error ($\varepsilon_t^{\overline{\Delta y}/4}$). This model specification allows for shocks in both the potential output and the trend GDP growth rate. These shocks, ($\varepsilon_t^{\bar{y}}$) and ($\varepsilon_t^{\overline{\Delta y}/4}$) respectively, are serially uncorrelated and mutually contemporaneously uncorrelated with (θ_t) and (ε_t^{θ}). The remaining equations (Eq. 7-9) represent the other remaining transition equations of the state-space model.

4 – ESTIMATION METHODOLOGY

4.1 Prior Distributions

We introduce prior parameter means, modes and standard deviations in order to initialise the KF and to evaluate the likelihood function. The distribution moments are set-up in such a way as to allow a large portion of the probability density to concentrate around the prior mean and mode. This depends inherently on how strong our prior belief is on the position of the true parameter value in the parameter space (An & Schorfeide (2007)). In addition, the proposed prior densities of several parameters are truncated. We do this to restrict the probability distribution in tail regions, where a resulting parameter estimate would be economically trivial (Lubik & Schorfeide (2007)).

Table 1 – Prior choices

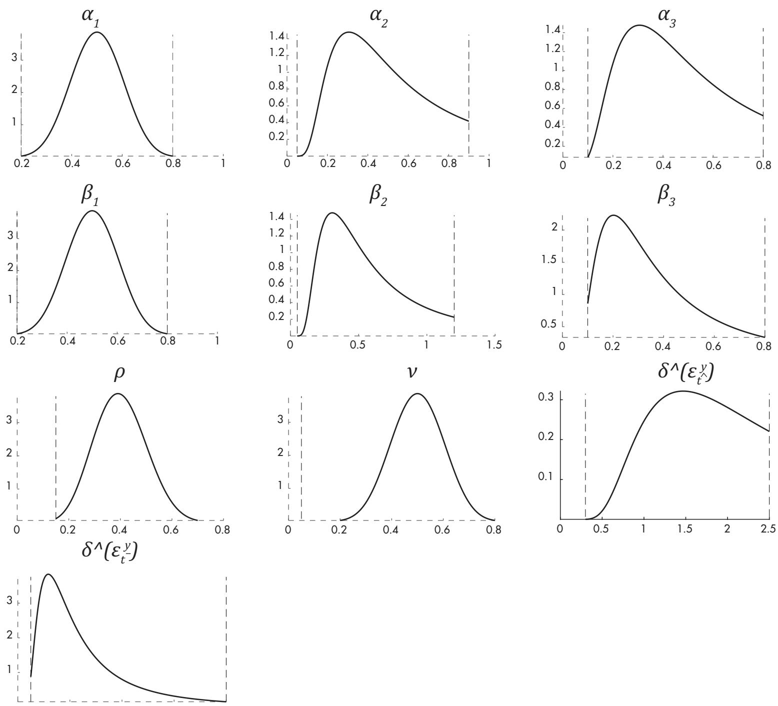
Estimated Parameters					
		Prior Distributions			
Parameter		Type	Prior	Mode	Std
Output Gap Persistence	α_1	Beta	0.50	0.50	0.10
RIR Deviation Channel	α_2	Inverse Gamma	0.40	0.30	5.00
REER Channel	α_3	Inverse Gamma	0.40	0.30	5.00
Inflation Persistence	β_1	Beta	0.50	0.50	0.10
MC - Output Gap	β_2	Inverse Gamma	0.30	0.30	5.00
MC - REER Gap	β_3	Inverse Gamma	0.20	0.20	5.00
Trend Growth Channel	ρ	Beta	0.50	0.39	0.10
Preferences Persistence	ν	Beta	n/a	0.50	0.10
Standard Deviations					
Output Gap	$\delta\^(\varepsilon_t^{\hat{y}})$	Inverse Gamma	1.00	1.47	10.00
Trend GDP	$\delta\^(\varepsilon_t^{\bar{y}})$	Inverse Gamma	0.25	0.12	5.00

The choice of initial priors for the parameters and standard deviations is based on those calibrated in the Medium-Term Projection Model (MPM) in use at the Bank of Albania (Hledik, Kika, Mitre (2021)), but with some subtle differences[5]. We have initially set the exchange rate and the interest rate channels' expected elasticities

[5] *The priors on real monetary conditions' elasticities ($\beta 2$, $\beta 3$) in the IS curve are higher compared to the calibrated values in the MPM, reflecting the more limited number of identifying components in the equation.*

on the output gap to be the same, to test whether real domestic and external monetary conditions are equally important in defining business cycle fluctuations. Priors on real marginal costs reflect the belief of a stronger expected effect of domestic price pressures (α_2) on inflation compared to foreign price pressures (α_3). We have set-up all prior means and modes for persistence parameters (α_1, β_1) to 0.5, bar the persistence of the preference variable (ν). Here, there is a lack of prior information on where this parameter should fall into and, as the variable is determined through the relationship of two unobserved variables, it is impossible to run initial naïve estimates through simpler econometric or statistical techniques. We expect the signal to noise ratio of the output gap ($\delta\hat{}(\varepsilon_t^{\hat{y}})$) to be around 4 times higher than that of the level of potential GDP ($\delta\hat{}(\varepsilon_t^{\bar{y}})$), with the belief that there is also some volatility in potential GDP. Volatility of the latter will be important in allowing the trend GDP growth rate, the component with the lowest frequency, to be estimated.

Figure 2 – Prior distributions and bounds



Source:

Priors on autoregressive parameters are set to follow a beta distribution. Prior distributions are bounded on most persistence parameters in order to prevent the optimizing algorithm from crashing at values equal to 0 or 1. For behavioural parameters of variables in gap form entering the IS and Philips curve measurement equations, we have selected an inverse gamma distribution, in order to skew the probability density around the prior mean and mode, but to also lower the chances of obtaining large and economically insignificant parameters. Again, we truncate the probability distribution of parameters measuring the effects of marginal cost components and monetary conditions. While there are no set theoretical limits on where to expect the true value of such parameters and results should be subject to economic idiosyncrasies of each country (Amato (2005)), we do not want the ML estimation to converge into implausible and unrealistic parameter estimates. Prior probability distributions on standard deviations are also set to follow an inverse gamma distribution.

4.2 Calibrated Coefficients

Parameters guiding the dynamic properties of observed variables in the system are calibrated. This is done to ensure parsimony in estimation and to allow for the efficient optimisation of the key parameters of interest. Persistence parameters (γ_1,γ_2) of the real interest rate and real exchange rate gap, treated as observed variables within the system and specified through simple AR processes, are calibrated to 0.7. Standard deviations of the shocks for the same variables ($\delta\hat{}(\varepsilon_t^r), \delta\hat{}(\varepsilon_t^{\hat{z}})$) are also calibrated to 0.7. Finally, the standard deviation of structural shocks ($\delta\hat{}(\varepsilon_t^{\Delta\pi})$) onto annual inflation is set at 3.5. A relatively higher standard deviation for the latter will allow the filter to efficiently remove excess noise from the variable and process effectively the signal component used to estimate the unobserved states. Here, we are ensuring that unnecessary excess noise does not induce a downward bias on parameter estimates and yield highly transitory and implausible unobserved states. Nevertheless, the calibration of the parameter will in no way be important in defining the overall shape and position in time of unobserved variables.

Table 2 – Calibrated coefficients

Calibrated Parameters		
Parameter		Value
RIR Persistence	γ_1	0.7
REER Gap Persistence	γ_2	0.7
Standard Deviations		
Annual Inflation	$\delta^\wedge(\varepsilon_t^{\Delta\pi})$	3.5
RIR	$\delta^\wedge(\varepsilon_t^{\bar{r}})$	0.7
REER Gap	$\delta^\wedge(\varepsilon_t^{\hat{z}})$	0.7

4.3 Data

Table 3 shows the list of observed variables used and their characteristics, subject to the model specification, and other auxiliary variables necessary for simple data transformation, used to obtain core model variables. Data are available for the majority of the variables from 1998q1. The policy rate is an auxiliary variable used to construct the short-term real interest rate and is available from 2001q2. The full data sample ends uniformly in 2017q4. Real GDP in levels and headline inflation are directly observed variables. The real exchange rate gap (REER), used as a proxy for relative price cycles, is constructed through removing the trend component of a simple REER variable through an HP filter. The REER variable is measured through a standard ratio of foreign and domestic CPI indices multiplied by the nominal exchange rate.

Table 3 – List of variables

Variables						
Name		Sample	Use	Form	Type	Source
GDP	y_t	1998q1-2017q4	Main model	Level (log)	Observed variable	INSTAT
Headline Inflation	$\Delta\pi_t$	1998q1-2017q4	Main model	Annual change (%)	Observed variable	INSTAT
Real Exchange Rate Gap	$\hat{z}_t$	1998q1-2017q4	Main model	HP Filter (p.p. to HP trend)	Observed variable	Author's calculations
Real Interest Rate	r_t	2001q2-2017q4	Main model	Level (%)	Observed variable	Author's calculations
Domestic CPI		1998q1-2017q4	RER gap derivation	Level (log)	Auxiliary variable	INSTAT
Nominal Exchange Rate		1998q1-2017q4	RER gap derivation	Level (log)	Auxiliary variable	Bank of Albania
Foreign CPI		1998q1-2017q4	RER gap derivation	Level (log)	Auxiliary variable	EUROSTAT
Policy Rate		2001q2-2017q4	RIR derivation	Level (%)	Auxiliary variable	Bank of Albania

Source: Bank of Albania, INSTAT, EUROSTAT, author's calculations

4.4 Parameter Restrictions – Three-Step Process

After setting up parameter priors and probability distributions and brining the model to the data, we can transform the model into state-space form, run the KF to estimate the likelihood function and maximise it through the ML technique. Nevertheless, the relative contribution stemming from the ML estimated standard deviations of trend growth ($\varepsilon_t^{(\overline{\Delta y}/4)}$) and of natural rate of interest (ε_t^{θ}) to the overall model variability would prove to be relatively small and their values biased towards zero, known as the so-called "pile-up problem" in Stock (1994)[6]. After trying to estimate the model with the priors chosen, both of these parameters were estimated to be zero. In order to tackle such issue, Stock and Watson (1998) median-unbiased estimators were used, by introducing two restrictions to the model:

$$\lambda_g = \frac{\delta^{\varepsilon_t^{\frac{\overline{\Delta y}}{4}}}}{\delta^{\varepsilon_t^{\bar{y}}}} \qquad \lambda_z = \frac{\delta^{\varepsilon_t^{\theta}}}{\delta^{\varepsilon_t^{\hat{y}}}} * \frac{\alpha_2}{\sqrt{2}}$$

In order to be used as an input in the λ_g calculation process, a preliminary estimate of potential output ($\bar{y}_t$) is obtained, following Kuttner (1994). The KF is used on a simplified model of equations which: (i) ignore the real interest rate channel from the IS curve and (ii) assume the trend GDP growth rate ($\overline{\Delta y}_t/4$) to be constant.

$$\hat{y}_t = \alpha_1 * \hat{y}_{t-1} + \alpha_3 * \hat{Z}_{t-1} + \varepsilon_t^{\hat{y}} \qquad (1)$$

$$\overline{\Delta y}_t/4 = cte \qquad (6a)$$

As in Andrews and Ploberger (1994), the exponential Wald statistic for a structural break with unknown date is computed on the preliminary potential output growth series ($\Delta\hat{y}_t$) and the Stock and Watson (1998) table is used to convert the Wald statistic into a median-unbiased estimator.

The second step consists in estimating another series for potential output to help in generating an unbiased estimate, λ_z. After using the newly found value of λ_g to measure an unbiased estimate of

[6] *The large majority of the probability piled up at zero in the probability density function.*

the trend GDP growth rate's signal to noise component ($\varepsilon_t^{\overline{\Delta y}/4}$), the KF is applied to a model with the following characteristics (i) the real interest rates channel in the IS curve is included and (ii) the preference variable (θ_t) assumed to be constant. The Wald statistic for an intercept shift in the demand equation at an unknown break date is then computed on the newly generated series of the trend GDP growth rate ($\overline{\Delta y}/4$) and then converted to a median-unbiased estimator λ_z. The third and final step constitutes in estimating the remaining model parameters by imposing the abovementioned values of λ_g and λ_z, as restrictions on parameter estimates, so that:

$$\delta^{\varepsilon_t^{\frac{\overline{\Delta y}}{4}}} = \lambda_g * \delta^{\varepsilon_t^{\bar{y}}} \; ; \quad \delta^{\varepsilon_t^{\theta}} = \frac{\lambda_z * \delta^{\varepsilon_t^{\hat{y}}}}{\alpha_2} * \sqrt{2}$$

The λ estimates point to intercept breaks in both the trend GDP growth rate ($\overline{\Delta y}_t/4$) and the natural rate of interest ($\tilde{r}_t$). Confidence intervals at 90% provide further ground for checking the robustness and variations in the estimation of the natural rate of interest ($\tilde{r}_t$) in time.

Table 4 – First and second step estimation results

Variable	Median-unbiased Estimate of λ		
	Point Estimate	90% Confidence Interval	
		Low	High
Trend growth	0.33	0.00	0.41
Natural rate	0.40	0.22	0.79

4.5 Maximum Likelihood Estimation

We follow a ML estimation strategy involving several steps to estimate the model parameters and the unobserved variables. In addition, we employ a sampling algorithm to infer the marginal posterior distribution of parameter estimates. With the model transformed in linear state-space form and solved for its unique solution, we combine the data with the priors, with the calibrated coefficients and with the imposed restrictions on the signal-to-noise ratios in order to obtain the minimum of the minus log-likelihood function.

Next, we run an optimisation algorithm that finds the minimum of a constrained multivariate objective function ("Fmincon") to obtain maximized parameter means in the parameter space at the optimum (Han (1977); Powell (1978)). The estimation period chosen is 2005 to 2017. The optimisation routine is constrained by the bounds specified on the proposed prior distributions of parameters. The optimisation algorithm is convenient in that it allows to easily calculate the contribution of data and of priors onto the minimization of the objective function and derivation of parameter estimates, by extracting the specific values contained in the Hessian information matrix of second derivatives (Kiley (2015)). In addition, we check the local behaviour of the objective function around the estimated parameter values so that we can ensure the minimization routine has reached a local minimum in the immediate neighbourhood of the parameter space. This is done to confirm that the likelihood function is not flat at this specific point and other parameter estimates are not potentially available (An and Schorfeide (2007)) (results in Annex).

Posterior parameter estimates will be proportional to the likelihood function of the data and the priors, but up until this step there is no knowledge on the target posterior probability distributions since they are analytically intractable. To simulate target posterior marginal densities of parameter estimates, we use a Monte-Carlo Markov-Chain (MCMC) technique implemented through an adaptive random walk Metropolis-Hastings (MH) algorithm[7]. This sampler allows for draws from the targeted posterior to be conditional on the full accumulated MCMC chain while automatically "learning" and adapting better parameter values of the algorithm while running (Haario, Saksman & Tamminen (2001), Roberts and Rosenthal (2009)). Posterior pseudo-estimates are used to initialize the sampler. The convergence of the MCMC chains around posterior parameter estimates will be largely conditional on the information the data brings onto the likelihood function for each parameter and, additionally, on the information each prior bears that is not contained in the data. We set the number of draws at 1 million, in order to allow the MH algorithm enough room for the generated MCMC chains to converge and to explore sufficient

[7] *The MH sampling algorithm was first introduced in Metropolis et al. (1953) and then refined and expanded for a large range of possible Monte-Carlo sampling methods by Hastings (1970).*

points of the targeted posterior distributions. The latter will allow a direct comparison between prior and posterior probabilities and provide valuable insights on the amount of information data bring to specific parameter estimates. The acceptance ratio is set at 0.234. Setting the acceptance ratio at this level, on the one hand avoids the sampler having a long memory and fail to converge, but on the other hand allows for sufficient movement for the exploration to efficiently sample from the target distribution (Bedard (2008), Sherlock and Roberts (2009)). Following standard practices with sampling algorithms, we allow for a burn-in ratio of 20% of the MCMC chain with the assumption that convergence of the chain to its stationary state has not been reached up to this point (Roberts & Casella (1999)) (MCMC chains in Annex).

As a final step, we run a two-sided KF to obtain the full sequence of unobserved states conditional on estimated parameters. We generate sequentially the filtered states of unobserved variables up to the last quarter of available data and thereafter, we use the Kalman Smoother to obtain an update of all unobserved states conditional on all previous information. The filter is run for the full data sample from 1998, but we report results starting from 2001 as convergence of estimates for unobserved states is reached at about this period.

5 – RESULTS

5.1 Parameter Estimates

Table 5 shows the parameter estimates of the system. We report posterior modes and associated parameter uncertainties, in line with works following a similar estimation methodology (An & Schorfeide (2007); Bjornland, Leitemo & Maih (2010); Lubik & Mathes (2015)). Parameter estimates largely yield expected point estimates and fall within reasonable and significant probability intervals. Estimates reject prior beliefs for stronger combined effects of real monetary conditions on the IS curve. The elasticities of the real interest rate (α_2) and of the real exchange rate channels (α_3) are centred at respectively 0.22 and 0.15. While lower than expected, these results confirm the importance monetary policy has onto smoothing cyclical fluctuations directly, through real interest rates, and indirectly, through affecting the nominal exchange rate and consequently the real exchange rate. Real marginal cost parameter estimates (β_2,β_3) indicate a stronger influence on consumer prices from domestic relative to foreign pressures. The result is expected and intuitive as the core component of the CPI inflation has a high share and should be more immune to foreign price fluctuations. However, the point estimates can be less precisely inferred, reflecting the larger probability interval of the estimates at the margins of statistical significance. Posterior estimates for persistence parameters of both lagged output gap (α_1) and inflation (β_1) are around 0.6. While slightly higher than prior expectations, they allow for enough variation in observed data to be explained by unobserved variables of interest, in this case the natural rate of interest ($\tilde{r}_t$) and the output gap ($\hat{y}_t$), and ensure that their behaviour in time can be estimated. The estimate of the elasticity of the natural rate of interest (ρ) to trend GDP growth rate is 0.45. The trend GDP growth rate ($\overline{\Delta y}_t/4$) is a determinant of the level of the natural rate of interest ($\tilde{r}_t$), whereas the behaviour of the preference variable (θ_t) in time is a determinant of the variation around this level. The autoregressive coefficient (ν) of the latter is 0.5, equal to the mode of the prior distribution.

Table 5 – Parameter estimation results

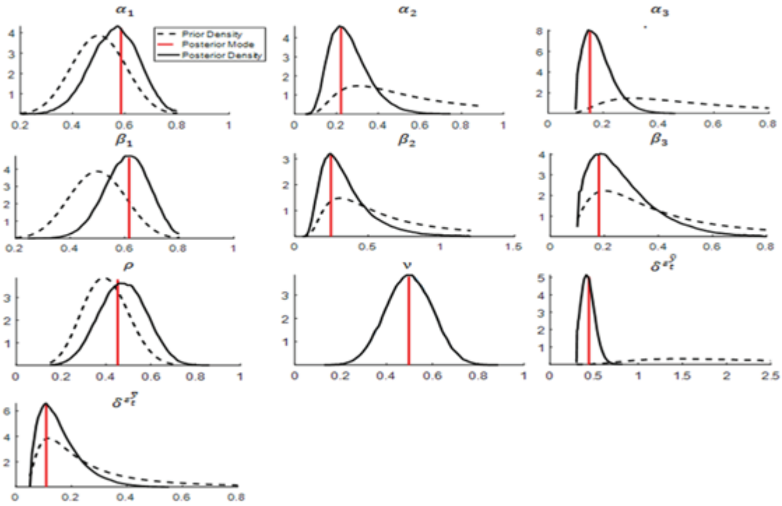
Estimated Parameters							
		Prior Distribution			ML Estimation Results		
Parameter		Type	Mean	Mode	Point Estimate	Estimate Std	90% prob. Int.
Output Gap Persistence	α_1	Beta	0.50	0.50	0.59	0.09	[0.44-0.68]
RIR Deviation Channel	α_2	IG	0.40	0.30	0.22	0.09	[0.15-0.38]
REER Channel	α_3	IG	0.40	0.30	0.15	0.05	[0.12-0.26]
Inflation Persistence	β_1	Beta	0.50	0.50	0.62	0.08	[0.49-0.71]
MC - Output Gap	β_2	IG	0.30	0.30	0.25	0.18	[0.17-0.59]
MC - REER Gap	β_3	IG	0.20	0.20	0.18	0.12	[0.14-0.45]
Trend Growth Channel	ρ	Beta	0.50	0.39	0.45	0.10	[0.34-0.61]
Preferences Persistence	ν	Beta	n/a	0.50	0.50	0.10	[0.37-0.63]
		Standard Deviations					
Output Gap	$\delta\hat{}(\varepsilon_t^{\hat{y}})$	IG	1.00	1.47	0.44	0.06	[0.34-0.50]
Trend GDP	$\delta\hat{}(\varepsilon_t^{\bar{y}})$	IG	0.25	0.12	0.11	0.06	[0.07-0.23]
Trend GDP Growth*	$\delta\hat{}(\varepsilon_t^{\Delta\bar{y}/4})$				0.04		
Preferences**	$\delta\hat{}(\varepsilon_t^{\theta})$				0.55		

* Conditional on λ_g.

** Conditional on λ_z.

The signal to noise ratio of the output gap ($\delta\hat{}(\varepsilon_t^{\hat{y}})$) relative to trend GDP ($\delta\hat{}(\varepsilon_t^{\bar{y}})$) is at the expected minimum of 4. The information in the data does not seem to support more variability in the output gap relative to the trend component of GDP. In addition, trend GDP is about 3 times more volatile than its low frequency underlying structural trend growth rate ($\delta\hat{}(\varepsilon_t^{\Delta\bar{y}/4})$), conditional on the level of λ_g. The estimate of the preference variable's standard deviation ($\delta\hat{}(\varepsilon_t^{\theta})$) indicates that it has the highest variability in time relative to the other unobserved components. The latter is intuitive considering the inclusion of this latent variable in the natural rate of interest equation to work as an aggregation of several different sets of preferences along the business cycle, which can at times be concurrent or antagonistic.

Figure 3 – Posterior distributions



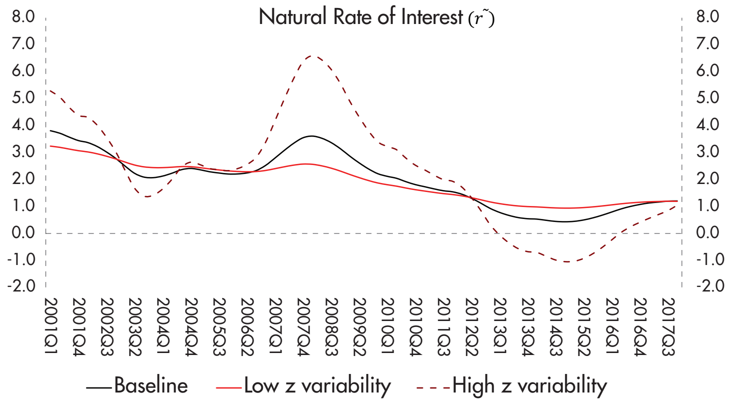
In Figure 3, we report the posterior distributions of parameter estimates obtained from the MH simulation. The shapes of the simulated target marginal densities are generally well defined and peak at the maximised parameter values. These characteristics are important in that they establish the ability to identify the posterior probability distributions and the absence of other candidate maxima for parameter estimates. While MCMC chains converge at different speeds for different parameters, they all achieve stationarity at the end of the sampling routine (see Annex). Information obtained from the Hessian information matrix of second derivatives shows that data contribute around 82% to the maximisation of the likelihood function. Thus, the information data bring is important in updating the priors, and posterior distribution moments are expected to be different from the proposed priors.

Marginal posterior distributions for autoregressive parameters (α_1,β_1) are tighter around the parameter estimates and shifted to the right. We obtain a similar result from the simulation of the posterior distribution of the parameter estimate of the elasticity of trend GDP growth rate on the natural rate of interest (ρ). The posterior distributions of the IS curve's structural parameters (α_2,α_3) are centered on a lower value in the parameter space and the densities are tightly concentrated around the parameter point estimate. Here, posterior probability simulation does not support the large dispersion proposed in the prior distributions. This result is even more extreme for the marginal posterior distribution of the output gap standard deviation (δ^($\varepsilon_t^{\hat{y}}$)). Information borne in the data rejects the large dispersion proposed in the prior. Posterior distributions for the real marginal cost parameter estimates in the Philips Curve (β_2,β_3) and for the standard deviation of trend GDP (δ^($\varepsilon_t^{\bar{y}}$)) have a slightly larger density around their maxima and shorter tails compared to the proposed prior distributions. There is no information in the data on the posterior distribution of the preference variable's autoregressive coefficient (ν) as the MH simulation returns the prior distribution. This result is intuitive considering the preference variable is a function of two other latent variables with no direct link to any measurement equation.

5.2 Kalman Filter Estimates of Unobserved variables

We apply the KF to generate all the states of unobserved variables using the parameter estimates to fully identify the model. The KF is run from 1998 to end-2017 but we report results starting from 2001 so as to allow the filter to converge and discard uninformative results prior to convergence. Besides the baseline estimation subject to point parameter estimates, we simulate two other versions of the model with low and high λ_z variability (see Table 4). In the first version, the volatility of the preference variable is at its lowest, and conditional on the minimum value of parameter estimates for the standard deviation of output gap (δ^($\varepsilon_t^{\bar{y}}$)) and the parameter estimate for the real interest rate channel in the IS curve (α_2) within the 90% confidence interval. The opposite is true for the second version. The simulations of unobserved states through these alternative parameterizations are useful in illustrating the uncertainty surrounding the estimation natural rate of interest and other unobserved variables in time.

Figure 4 – KF estimates of the natural rate of interest



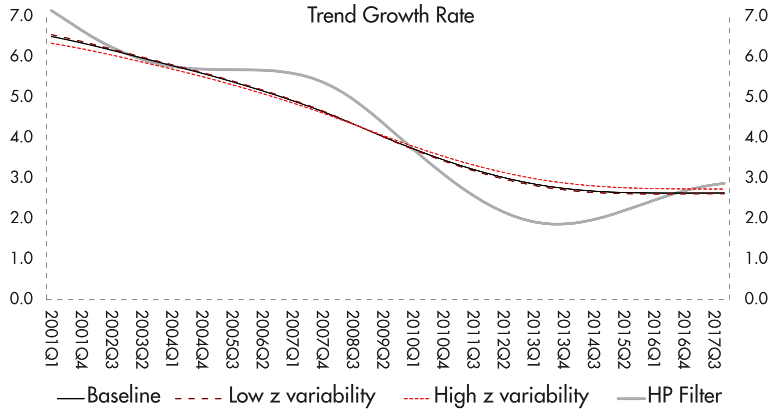
The natural rate of interest in Albania (Figure 4) has experienced a structural break at around the time of the global and financial crisis of 2008, present both in the baseline estimation and in the low and high variability versions of the estimates[8]. The break in the series is more evident in the high variability estimates and less so in the baseline and the low variability simulation. The pre-crisis average natural rate of interest is about 2.8% in the baseline version of the model and 3.6% and 2.6% in the high and low variability versions. Post-crisis means range from 1.2% in the baseline model to 1.0% and 1.3% in the high and low variability versions. End-point estimates range between 1.05-1.2%.

The structural break in the natural rate of interest is a consequence of the marked deceleration in the trend GDP growth rate (Figure 5), which is a direct and sole factor in establishing the level of the natural rate of interest in time. The trend GDP growth rate is estimated in this model to have moved from an average range of about 5.4-5.5% during the pre-crisis years, the 2001 to end-2008 period, to about 3.0-3.1% post-crisis and is estimated to have followed a steady declining path from the initial point of estimate in 2001. Trend GDP growth rate dynamics in Albania are estimated to have reflected the sharp deceleration in TFP growth (Çeliku et al. (2018)). The latter

[8] *See Section 4.4 for a more detailed description on the empirical estimation results of the structural breaks in the trend growth and natural interest rate series.*

was exacerbated by the 2008 global economic and financial crisis that exposed the underlying structural weaknesses of the Albanian economy and the need for reforms to prop-up the potential growth of the economy (IMF (2016)).

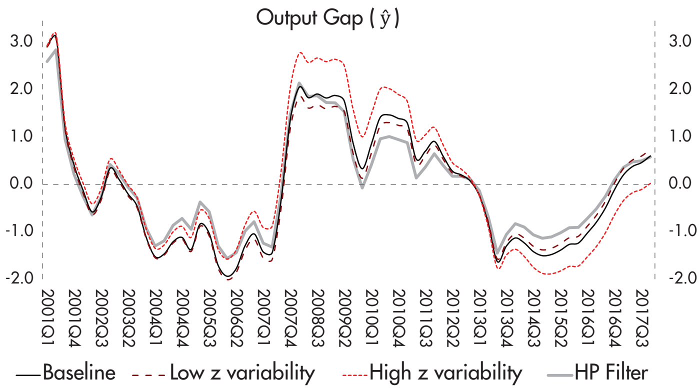
Figure 5 – KF estimates of the trend GDP growth rate



The natural rate of interest reaches its maximum during the last quarter of 2007 and first quarter of 2008, conditional on the parameterisation of the model. Estimates range from 3.6% in the baseline to 6.6% and 2.6% in the high and low variability versions. On the other hand, the minima are reached in the last quarter of 2014 and they range from 0.4% in the baseline model to -1.05% and 0.9% in the high and low λ_z variability versions.

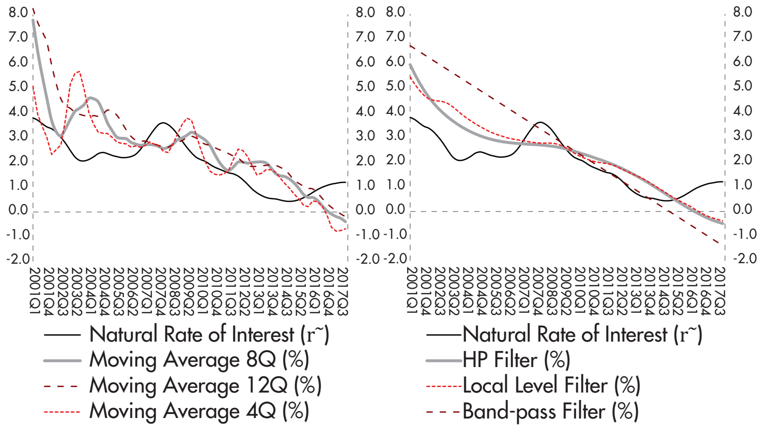
Fluctuations of the natural rate of interest around its level are a direct function of the variability of the preference variable in time. Since the latter aggregates different interest rate preferences along the business cycle, necessary to bring the economy back to its potential, its variation in time is conditional on the estimate of the output gap (Figure 6). The points where the natural interest rate is at its highest and lowest levels reflect the peak and trough of the business cycle in Albania. A distinction needs to be made here on the role of the cyclical position of the economy on the level of the natural interest rate. In the pre-2008 period, trend GDP growth rates were still relatively high and the average natural interest rate ranges from 2.4% in the baseline and the low λ_z variability estimates to 2.5% in the high λ_z variability version. After 2008, where the trend GDP growth rate is in a secular decline, the estimate of the average natural interest rate ranges from 0.7% in the baseline model to -0.3% and 1.0% in the high and low λ_z variability versions. The estimated pick-up in the natural interest rate since at least mid-2015 is attributed to the improvement of the cyclical position of the economy while the trend GDP growth rate is estimated to have remained rather flat. These results indicate the importance of both the rate of trend GDP growth and of the position in the business cycle of the economy in defining the level of the natural rate of interest at any point in time.

Figure 6 – KF estimates of the output gap



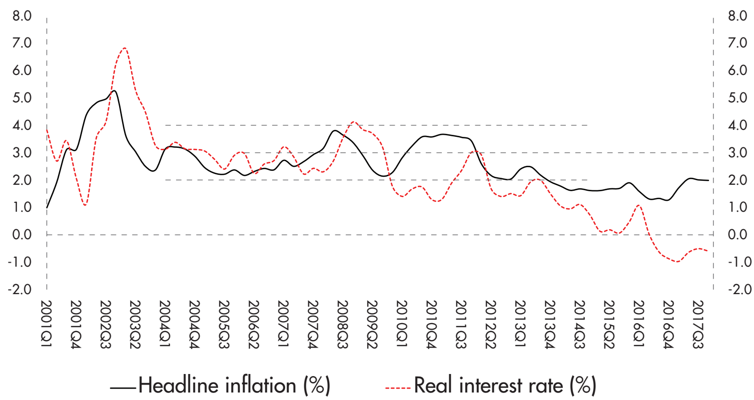
Simple moving averages over a range of quarters and in the form of univariate time-series filters on the real interest rate variable can provide alternative but more simplistic approaches to get a glimpse on the possible position of the natural rate of interest over time (Figure 7). However, one has to bear in mind that the outcomes of such exercises are fully conditional on the value of headline inflation and on the policy rate at every time period. As such, these measurements can refer the current stance of monetary policy but cannot provide inference as to where the natural rate of interest would be if equilibrium conditions for the economy and for inflation were to be met at every point in time.

Figure 7 – Simple moving averages (left) and univariate filter estimations (right) of the real interest rate



Consequently, if alternative approaches were to be selected, the estimation would be prone to errors when there are large fluctuations of demand and supply factors and when there is inflationary or deflationary pressures (Figure 8). These errors, while subject to the implicit monetary policy decision making process with respect to the form of inflation target, could nevertheless give an inaccurate view of the neutral monetary policy stance and the intensity of monetary policy impulse.

Figure 8 – Real interest rate, headline inflation and the target



Our findings on the behaviour of the natural rate of interest throughout the past two decades are in line with other research on developed countries, developing countries in the CEE and the immediate SEE region. All results share a commonality, in the form of declining potential GDP growth rates and natural interest rates. For developed countries, these trends were present well in advance of the global economic and financial crisis, as evidenced in the original research from Laubach and Williams (2003) and in other related works (Manrique and Marquez (2004), Benati and Vitale (2007), Holston et al (2016) and Zhu et al (2016)). In the CEE and SEE countries the decline in the natural rate of interest is a more contemporary phenomenon, subject to the structural break in the potential GDP growth rate in the aftermath of the global economic and financial crisis (Grafe et al. (2018); Hledik and Vlcek (2017); Us (2018), Grui et al. (2018); Baksa et al. (2013); Stafanski (2017); Acatirnei et al. (2018)). The results for the CEE and SEE region differ from our results on Albania only in terms of the point in time and the speed of recovery of the natural rate of interest, where present, or the trend of the natural rate at the end of the estimation sample, where increase in natural rates is not present. Natural rates of interest have been estimated to be edging upwards from their minima in Czech Republic, Hungary and Romania and stabilised at low levels in Turkey, Ukraine and Poland. Our results are similar to those in the first group of countries, in terms of the recovery of the natural rate of interest. However, they differ with respect to the speed of the recovery in natural interest rates, since our estimate of the pick-up in the potential GDP growth rate is much slower.

6 – MONETARY POLICY IMPLICATIONS

The evolution of the natural rate of interest in time is a vital information component in the monetary policy decision making process. While here we introduce results based on one estimation methodology and other alternatives are possible and necessary, the relevance and use of the results can guide policymakers accordingly. The uncertainty surrounding the estimation and the time-varying nature of the natural rate of interest poses challenges to the conduct of monetary policy as it affects the appropriate monetary policy stance and the design of an optimal policy reaction (Laubach and Williams (2015)). Additionally, misconceptions on both the level and the evolution of the natural rate of interest can lead to second-best economic outcomes for which the policymakers will not necessarily have a priori information (Orphanides and Williams (2002)). These misconceptions can lead to policy errors, which in hindsight can be costly, despite policymaking following a clear and well-established rules-based strategy (Yellen (2012)). The effects of these policy errors can be diverse and not easily discernible from hard data or economic outcomes for a considerable amount of time. Sub-optimal monetary policy decision making can delay economic stabilization through allowing for persistent inefficiency gaps and increasing the variability of inflation (Barsky, Justiniano and Melosi (2014)).

Our attempt at pinning down the behaviour of the natural rate introduces a new tool to assess the evolution of the implicit neutral policy rate at different points in time and to complement other methodologies in place. While the estimate of the natural rate of interest for Albania proposed here is likely to be imprecise and subject to a considerable degree of uncertainty, knowledge about its approximate real-time evolution is an additional layer of assistance to the conduct of monetary policy over the business cycle and with respect to the inflation target. As such, the policy-maker can at any time evaluate the adequacy of the monetary policy stance, the intensity of the current monetary policy impulse and the intensity of the future proposed changes in the monetary policy impulses with respect to achieving the inflation target within a pre-defined time horizon.

The aim of this tool is not to provide a framework for monetary policy evaluation exercises in Albania but rather to reduce the uncertainty linked to the forward-looking nature of the conduct of monetary policy. The former is a challenging empirical task and subject to a large degree of subjectivity in attempting to isolate and measure the impact of potential past policy errors on lingering economic inefficiencies. The likelihood of committing policy errors is nevertheless diminished when uncertainty about the approximate position of the natural rate of interest is removed from the decision-making process and a range of tools are employed to aide in the process.

Finally, the estimated decline in level of the natural rate of interest makes the probability of hitting and operating near the effective lower bound higher (Kiley and Roberts (2017)). With the decline in the natural rate of interest, as our results indicate, there is comparatively less room to manoeuvre for monetary policy in Albania, in cases of cyclical downturn or outright recession episodes. This makes the necessity to introduce unconventional monetary policy tools in Albania more likely and the estimation the natural rate of interest in the future ever more important, as an instrument that enables the policy-maker to exactly pinpoint the right moment when that policy space has fully ran out.

7 – CONCLUSION

The natural rate of interest is an important benchmark for evaluating the evolution of the natural rate of interest in time and in identifying the neutral monetary policy stance on which to anchor monetary policy decisions. Estimating its evolution to correctly inform policymakers on the true movement of the stabilizing equilibrium interest rate has become ever more important in a post-crisis setting characterized by sluggish economic growth and declining real interest rates. In this work, we endeavor to estimate the natural rate of interest in Albania following the methodology introduced by Laubach and Williams (2003). We account for the evolution of the trend growth rate in time, the primary structural factor that defines the level of the natural interest rate, and for households' time preferences throughout the business cycle, that defines the volatility around the structural level.

We employ an initial three-step statistical estimation routine to identify potential structural breaks in latent variables. We identify intercept shifts in trend growth and in the natural rate of interest and use this knowledge to introduce restrictions to the parameter matrix to be estimated. Next, we follow a ML estimation strategy, using Kalman Filter to obtain the likelihood function, and the "fmincon" optimisation algorithm to obtain posterior modes that maximize this likelihood function. Parameter posterior probability distributions are approximated through an Adaptive Random-Walk Metropolis-Hastings algorithm and latent variables estimated through Kalman Filter.

We find that the natural interest rate has decelerated post-crisis due to slower trend growth and a downward shift of preferences on interest rates. The natural rate of interest is estimated to have reached its maximum value in the immediate year prior to the global financial crisis, in 2007, and its minimum during the trough of the business cycle in Albania in 2014. In the later period, the cyclical improvement estimated to have started from 2015 has induced a slight pick-up in the natural rate of interest. Despite, the estimation of the natural rate of interest in Albania being subject to a large degree of uncertainty due to its time-varying nature, this is a first attempt to pinning down its behaviour in time. As such, the results here provide an important empirical tool to enable the correct formulation of monetary policy along the business cycle and ensure a proper evaluation of its course and intensity.

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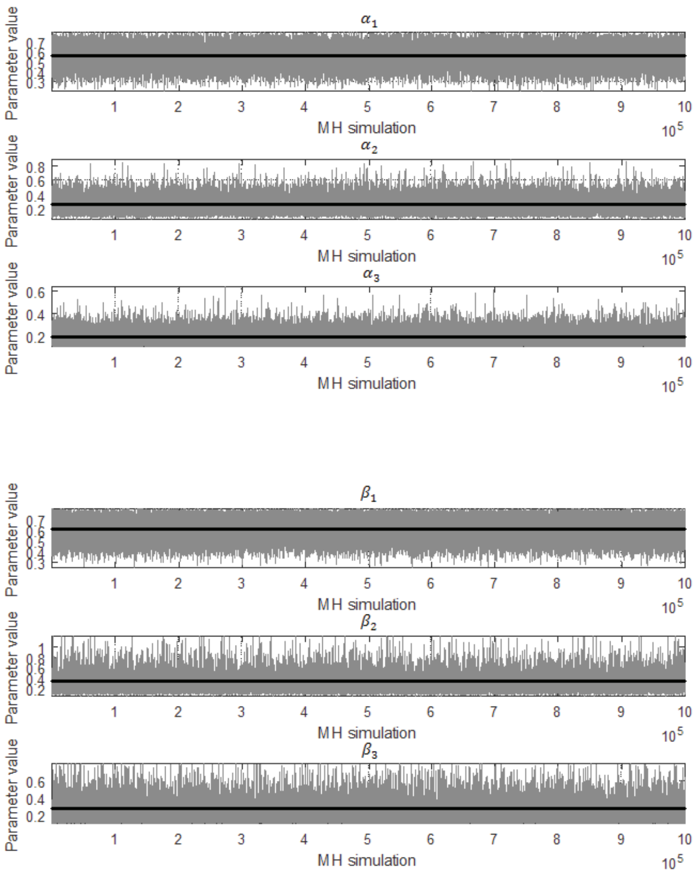
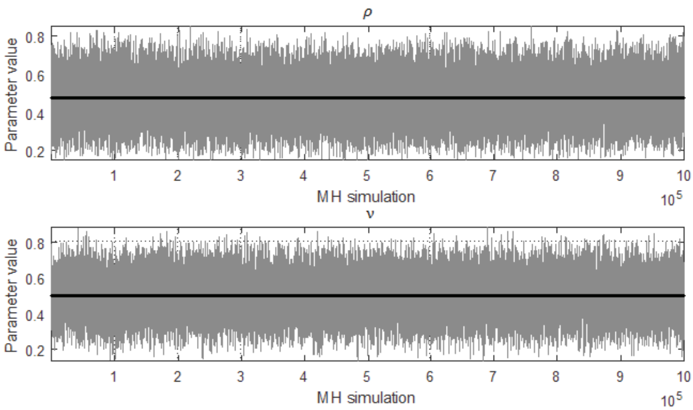
ANNEX

Figure 9 – MCMC Chains Trace Plots




ρ
Parameter value
MH simulation
10^5
ν
Parameter value
MH simulation
10^5

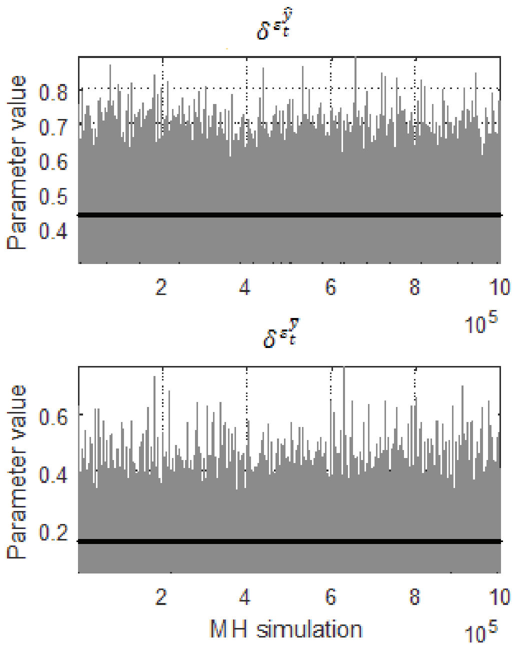
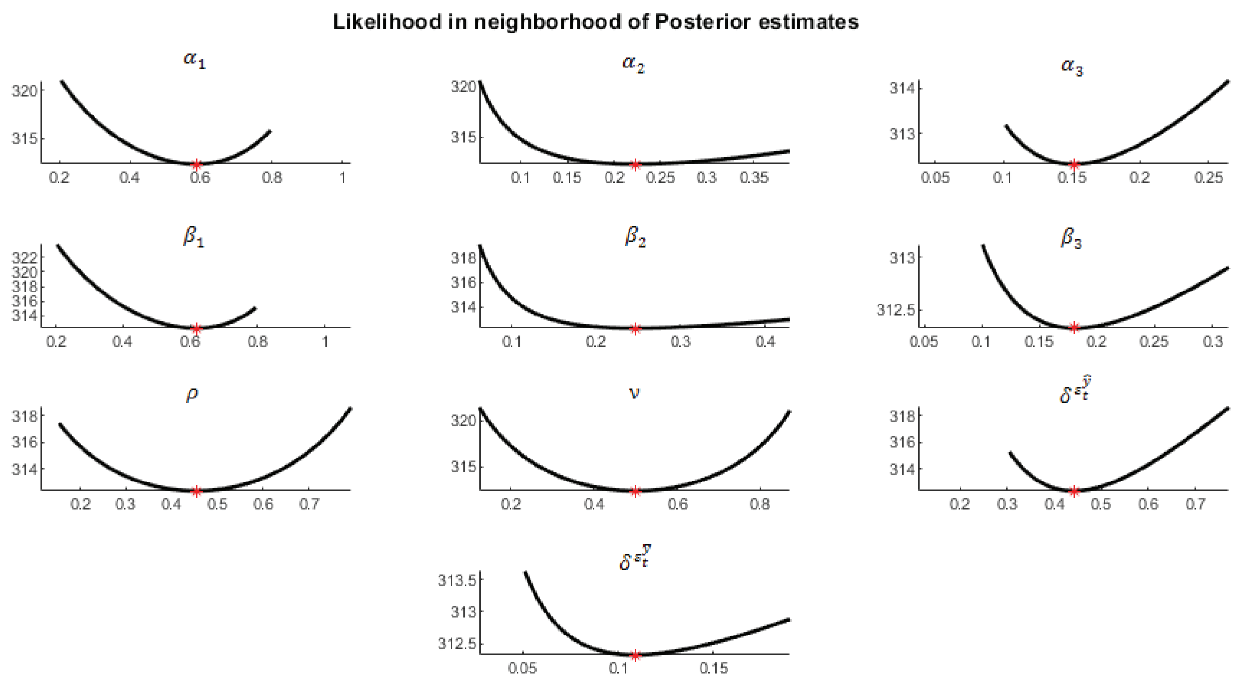

Parameter value
10^5
Parameter value
MH simulation
10^5

Figure 10 – Shape of Maximum Likelihood Function in the immediate neighbourhood of posterior parameter estimates



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