PRACTICAL ISSUES IN FORECASTING WITH VECTOR AUTOREGRESSIONS

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INTRODUCTION

Vector Autoregressive (VAR) models are widely used for forecasting economic indicators such as inflation, economic growth, and the exchange rate. They were proposed in the early 1980s by Sims (1980) as a more appropriate technique in the economic analysis compared to single models. Although the VAR method has encountered many criticisms and has been challenged over the years by many complex econometric techniques, it continues to be a reference or comparative method, due to the convenience and rapidity it provides when building economic scenarios.

The use of autoregressive vectors for forecast purposes raises some questions regarding the models' specification and evaluation. Generally, they are related to issues such as: a) selecting the appropriate number of time lags; b) the way of entering the statistical series in the model, in levels or differences, i.e. whether it is necessary for them to be stationary; c) model prediction strategy, if the extension of the number of observations gives us more information than a fixed and repeated sample (i.e. rolling), evaluated with a sufficient number of observations; d) the evaluation of the predictability within or outside a given period; as well as e) controlling for issues that arise from over-parameterization.

This paper addresses some of the practical issues encountered during the building of VAR models for forecasting purposes for the Bank of Albania. It relies on a model with several indicators and compares the forecast ability of its various specifications which can serve to potential users as a starting point for building their models. Firstly, we will discuss the models' evaluation and specification method, and later on the procedure followed for the forecast and the conclusions.

SPECIFICATION AND EVALUATION OF THE MODEL

Certainly, the specification of one model always depends on the purpose of its use. If a model is built to help with the monetary policy decisions, it should be able to predict key economic indicators that concern the Supervisory Council of the central bank, such as inflation and economic growth. VAR models that are commonly used in the studies for monetary transmission mechanism are generally based on a small set of indicators. For the purpose of this essay, I have used a small model with four domestic endogenous indicators – inflation, economic growth, exchange rate and key interest rate – and three exogenous foreign indicators – inflation, economic growth, and euribor in the euro area.

This set of indicators is in line with the structural model proposed by Svensson (2000) for small and open economies with inflation targeting regime.

The autoregressive vectors helps us determine the dynamics of the relationships that characterize the economic indicators in the model, and then use these estimates for their projection in the future. In the VAR method, the value of each indicator at the current time t is explained as the weighted average of past values of all series at time ***t-p*** plus a term that includes all other shocks at the current time. Mathematically, this would be expressed as:

$$y_t = c + B_1 y_{t-1} + \ldots + B_p y_{t-p} + u_t,$$

Where y_t shows the vector of variables included in the model, while u_t is the vector of errors measured as the divergence of observed values y_t from the forecast obtained from the linear combination of past values of y with the estimated parameters B and the constants c. The ability to forecast accurately is influenced by the values of the parameters used for their weighting, as well as by the number of time lags p of the observed series.

Parameter estimation using the least squares method in VAR models requires time series to be stationary, but many economists also use them in a non-stationary form. The presence of the trend on economic indicators and their sensitivity to the persistence of model errors can give spurious estimates of coefficients, so the stationarity test is important. However, the use of indicators in a non-stationary form is useful for capturing the cointegrating correlations, if they exist. For this reason, checking for the stationarity of variables should not be seen as mandatory, but as instructive to understand the dynamics of their qualities before the model's evaluation (Mahadeva and Robinson 2004).

With regard to determining the order of time lags in VAR, the literature offers several methods. They compare the performance of different specifications that take into account the size of the sample and the number of dependent variables. Some of them give priority to model efficiency by selecting the one that gives the smallest errors (such as the Final Prediction Error criteria (FPE), Akaike, and corrected Akaike), while others' main criterion is the consistency of the process of finding the real model (such as the Schwarz (BIC) and Hannan-Quin (HQ) criteria. Other studies have developed different approaches, such as focused information criterion, transfer function method, principle of predictable least square, combined information criterion, and so on, however AIC and BIC still remain the most popular methods (Ding, Tarokh and Yang, 2016). Selecting the "best" information criterion is difficult and requires a compromise, depending on what we want to optimize. Including a higher number of time delays decreases the degree of freedom in the model, and consequently the veracity of the estimated parameters. On the other hand, a small number of lags increases the likelihood of failing to capture some inter-temporal dynamics and the ability to remove autocorrelation in residuals (Lack, 2006).

Generally, the selection criteria of the BIC model from Schwarz suggests a spared number of lags; while the standard AIC criterion suggest numerous lags even for samples with relatively short periods. Asghar and Abid (2007) find that all the criteria reviewed by them may be valid for determining the real number of time delays, in case of regime alternations or system shocks; meanwhile, the authors recommend Schwarz BIC information criterion as the best for the models with large sample estimations. Similar to these authors, the simulation results from Ayalwe et al. (2012) show what BIC, HQ, Akaike-HQ median and BIC-HQ median may perform better in large samples, while the AIC-BIC combination median may be a reliable criterion in all small or large samples.

Concerns on the over-parameterization may push practitioners into using the most parsimonious BIC method in small sample models, however Liew (2004) finds that the more tolerant AIC and FPE criteria exhibit superiority against other criteria even in the cases of small samples (up to 60 observations). Furthermore, Hurvich and Tsai (1989) find that bias correction in the AIC method can increase the efficiency in small size samples, also when the proportion of the estimated parameters over the sample size is relatively large. Another attempt by Safi (2011) on the selection of autoregressive models under the presence of autocorrelation finds that "over-specification performs better in finding the true model, especially when the size of the sample is small compared to the number of estimated parameters" and that "the BIC criterion corrects the over-specification of AIC". In estimations with vector autoregressions, McQuarrie and Tsai (1998) state that the probability of overfitting the model is smaller than in multivariate regressions, despite the rapid increase in the number of parameters in VAR. Authors base this statement on the results derived from multiple simulations of VAR models, including large-scale estimations and small sample sizes. Therefore they recommend not to underestimate the problem of model under-fitting where heavy penalty functions can hinder the performance.

However, empirical researchers have shown that determining the number of parameters in VAR models is very important, especially if they are to be used for forecasting purposes. Loss of the degrees of freedom due to the high proportion of the number of coefficients compared to the number of observations may reduce the accuracy of estimated coefficients, thus weakening the predictive power (Wallis, 1989). Doan (1990) says that "predictions made with unrestricted vector autoregresions often suffer from model over-parameterization… (which) cause large out-of-sample forecast errors." There are several approaches that address this issue, imposing the value of coefficients, in order to reduce their uncertainty.

A common method used in the last two decades to solve the dimensionality problem is the Bayesian estimation, which consists in the shrinkage of the estimated parameters of the model, by setting some prior values. Unlike the traditional structural models where the overfitting is avoided by setting zero values for many coefficients (based on the theoretical preferences of modellers), the Bayesian method tries to achieve this by allowing the same number of parameters evaluated in VAR, and by reducing their sensitivity to

data. This way there is more flexibility in order to eliminate overfitting, allowing thus the representation of both, the preliminary economic expectations and the statistical estimates of the modellers (Todd, 1984).

FORECAST PROCEDURE

Earlier in the article we came across some of the most discussed issues for model building such as: transformation of time series into stationary form; choosing the number of time lags; and the use of the Bayesian method for avoiding the dimensional issue in VAR models. Table 1 shows the forecast procedure followed in this paper. The selected indicators (output, prices, exchange rate and interest rates) enter into our model in three forms. The variables in levels are intended to not circumvent a possible cointegrating relationship between the indicators, while their changes are intended to avoid spurious estimates that result in the case of a lack of cointegration to our non-stationary variables.

Table 1. Summary of the Forecast Procedure

Indicators	Forecasting Models	Estimation strategy	
		Recursive	Rolling
Level	VAR(1-4)		
	BVAR(1-4)		
YoY Differenced	VAR(1-4)		
	BVAR(1-4)		
QoQ Differenced	VAR(1-4)		
	BVAR(1-4)		

The data availability and the statistical noise that characterizes them during the first decade of transition impel us to narrow the exercise period for 2001-2017, with quarterly frequency. The data criteria for selecting the number of time lags in the model recommend for expansion of the information for more than one year (about 6-7 quarters according to AIC, HQ, FPE criteria); with the exception of Schwarz criterion, which suggests 3 lags for variables in first differences and 1 lag for the case where they are expressed as annual changes. Because the number of observations in our time frame is considered relatively small, the number of time lags of endogenous variables in VAR has been tested from 1 to 4 quarters for all estimates. Thus, the concentration of previous data within a year (with 1to 4 lags) limits the number of estimated coefficients (including exogenous ones) from 32 to 80. This numerical range is quite significant, especially if we narrow furthermore the evaluation period, to test the predictive ability of the model outside the evaluation period.

The forecasting experience with VAR models has shown that a model with good in-sample forecasts does not guarantee such satisfactory predictions in the out-of-sample period. Since, in practice, analysts involved in the economic policy proposal rely on forecasts for the future, the out-of-sample forecast ability of the model becomes more important. For this reason, the full available period, 2001Q1:2018Q1, is divided into so-called training periods, 2001Q1:2012Q4 (48 quarters), and in the forecast testing period,

2013Q1:2018Q1 (21 quarters). The number of coefficients evaluated in the model is considerable in relation to the number of observations. To control for the statistical issues in the OLS estimates due to possible over-parameterization, the forecast procedure with the unrestricted VAR model above has been repeated by using the Bayesian estimation method, BVAR. Determination of the prior parameters in the latter is carried out in a number of ways, but in this article I have followed a simple type, Normal-Wishart, as recommended by Carriero et al. (2011).

The exercise focuses on the predictive ability of the model in the short and medium term. The forecast evaluation is measured here by the root mean squared errors, RMSE, which compares the size of forecast errors through different estimations. More concretely, the procedure starts with the model estimation for the period 2001Q1:2012Q4 and for each of its specifications, according to the form of variables and the number of lags, a forecast is noted down for the 1, 4 and 8 quarters ahead. Moreover, the evaluation period recursively expands by a quarter, 2001Q1:2013Q1, calculating and maintaining the RMSE of forecasts for the time horizons we want. The evaluation process is repeated until 2017 Q4, when we are allowed to foresee in advance the last quarter 2018Q1. Apart from the recursive strategy, I have used and compared the rolling estimation as well. In this method, the training period is kept unchanged in a window of 48 observations, while the procedure for the re-evaluation and maintenance of RMSEs of the relevant specifications continues the same. Recursive and rolling strategies can improve linear model projections for an economy with continuous structural changes (Clark, 2008), so comparing their performance may serve to understand the existence of structural failure of economic indicators during 2001-2017, and if the loss of information due to short samples reduces the strength and validity of VAR models in Albania.

EMPIRICAL RESULTS

As noted in the beginning, this article does not aim to find the best forecast model but to discuss more about its nature. For this reason, the following analysis addresses the general characteristics of a good forecasting model for the Albanian economy. Table 2 shows the results on the predictive ability of the standard VAR model, according to the different expression forms of the variables and the choice of time lags. In order to have comparable RMSEs, despite the transformation of indicators, the gross domestic product, the price index and the exchange rate have been expressed as annual changes in percentage, while the key interest rate has remained unchanged. The highlighted figures in the table show the smallest errors, according to RMSE, of a specific compared vertically within its group.

Table 2. VAR Model: Average RMSE of Out-of-sample Recursive Forecasts, 2013Q1:2018Q1

Variables, in %	Annual Growth			Annual Inflation			RER, yoy chg.			Policy rate		
Forecast horizon	1Q	1Y	2Y	1Q	1Y	2Y	1Q	1Y	2Y	1Q	1Y	2Y
Variable transformation												
Levels	3.1	3.0	2.8	0.8	1.1	1.1	1.2	1.5	1.5	0.1	0.3	0.5
YoY	1.3	1.6	1.7	0.4	0.5	0.6	1.2	2.3	3.4	0.1	0.3	0.5
QoQ	2.3	2.9	3.0	0.7	0.9	0.9	0.8	1.3	1.8	0.2	0.3	0.5
Model selection												
1 Lag	2.5	3.0	2.9	0.8	1.1	1.1	1.0	1.7	2.2	0.1	0.3	0.4
2 Lags	2.9	2.9	3.0	0.7	0.8	0.8	1.1	1.6	2.2	0.1	0.3	0.5
3 Lags	2.2	2.4	2.2	0.6	0.7	0.8	1.0	1.7	2.2	0.1	0.3	0.5
4 Lags	1.4	1.8	1.9	0.5	0.6	0.8	1.0	1.8	2.3	0.2	0.3	0.5
Model selection: VAR in levels												
Level: 1 lag	3.4	3.5	3.0	1.2	1.8	1.7	1.3	1.5	1.3	0.1	0.2	0.3
Level: 2 lags	4.0	3.2	2.9	0.8	1.1	1.0	1.3	1.6	1.5	0.1	0.2	0.5
Level: 3 lags	3.7	3.4	2.9	0.7	0.8	0.7	1.1	1.6	2.0	0.1	0.4	0.6
Level: 4 lags	1.5	1.9	2.2	0.6	0.8	0.9	0.9	1.2	1.3	0.2	0.3	0.5
Model selection: VAR in annual changes (YoY)												
YoY: 1 lag	1.3	1.6	1.7	0.4	0.5	0.6	1.0	2.1	3.1	0.1	0.3	0.4
YoY: 2 lags	1.3	1.6	1.7	0.4	0.5	0.6	1.0	1.9	3.0	0.1	0.3	0.5
YoY: 3 lags	1.3	1.6	1.6	0.4	0.5	0.6	1.3	2.3	3.3	0.1	0.3	0.4
YoY: 4 lags	1.4	1.7	1.8	0.4	0.4	0.7	1.4	2.9	4.2	0.2	0.4	0.7
Model selection: VAR in first difference (QoQ)												
QoQ: 1 lag	3.0	3.7	3.9	0.8	1.0	0.9	0.8	1.4	2.1	0.1	0.3	0.6
QoQ: 2 lags	3.3	3.9	4.4	0.8	0.9	0.9	0.9	1.4	2.1	0.2	0.4	0.6
QoQ: 3 lags	1.7	2.1	2.1	0.7	0.9	0.9	0.7	1.2	1.4	0.2	0.3	0.5
QoQ: 4 lags	1.3	1.7	1.7	0.5	0.7	0.8	0.7	1.3	1.5	0.2	0.3	0.4

Table 3. Bayesian VARs: RMSE of Recursive Forecasts, in the Out-of-sample Period of 2013Q1:2018Q1

Variables, in %	Annual Growth			Annual Inflation			RER, yoy chg.			Policy rate		
Forecast horizon	1Q	1Y	2Y	1Q	1Y	2Y	1Q	1Y	2Y	1Q	1Y	2Y
Variable transformation												
Levels	6.7	6.0	4.6	1.3	1.4	1.2	1.5	1.5	1.3	0.1	0.2	0.3
YoY	1.4	2.0	2.6	0.3	0.5	0.5	0.7	1.1	1.4	0.1	0.3	0.5
QoQ	4.6	7.0	7.8	0.9	1.5	1.4	1.0	1.8	2.8	0.1	0.4	0.9
Model selection												
1 Lag	5.5	7.3	8.1	1.0	1.4	1.3	1.1	1.6	1.9	0.1	0.3	0.6
2 Lags	4.6	5.4	5.0	0.8	1.1	1.0	1.1	1.5	1.8	0.1	0.3	0.6
3 Lags	4.2	4.3	3.8	0.8	1.0	0.9	1.1	1.4	1.8	0.1	0.3	0.6
4 Lags	2.6	3.1	3.0	0.7	0.9	0.9	1.0	1.4	1.8	0.1	0.3	0.6
Model selection: VAR in levels												
Level: 1 lag	8.7	7.3	5.5	1.4	1.6	1.3	1.6	1.6	1.3	0.1	0.2	0.2
Level: 2 lags	7.3	6.8	5.1	1.3	1.5	1.3	1.5	1.6	1.3	0.1	0.2	0.3
Level: 3 lags	7.6	6.6	4.9	1.2	1.4	1.2	1.5	1.5	1.3	0.1	0.2	0.3
Level: 4 lags	3.0	3.4	2.9	1.1	1.2	1.1	1.3	1.3	1.2	0.1	0.2	0.4
Model selection: VAR in annual changes (YoY)												
YoY: 1 lag	1.4	2.0	2.7	0.3	0.5	0.5	0.7	1.2	1.5	0.1	0.3	0.6
YoY: 2 lags	1.4	2.0	2.7	0.3	0.5	0.5	0.7	1.1	1.4	0.1	0.3	0.5
YoY: 3 lags	1.4	2.0	2.6	0.3	0.5	0.5	0.7	1.1	1.3	0.1	0.3	0.5
YoY: 4 lags	1.4	1.9	2.6	0.3	0.5	0.5	0.7	1.1	1.3	0.1	0.3	0.5
Model selection: VAR in first difference (QoQ)												
QoQ: 1 lag	6.3	12.6	16.2	1.4	2.3	2.0	1.1	1.9	2.9	0.1	0.4	0.9
QoQ: 2 lags	5.0	7.4	7.4	0.8	1.3	1.2	1.0	1.6	2.7	0.1	0.4	0.9
QoQ: 3 lags	3.7	4.3	4.0	0.8	1.2	1.1	1.0	1.7	2.8	0.1	0.4	0.9
QoQ: 4 lags	3.3	3.9	3.5	0.8	1.2	1.1	1.0	1.7	2.8	0.1	0.4	0.9

Concerning the transformation of time series in the VAR model, the estimations with the data in annual changes turned out as the most preferred candidate for the forecasts of all variables, except the exchange rate. Individually, forecast errors by to this mean of expressing the indicators are considerably lower in the case of economic growth and inflation, while for the REPO rate the differences at the level or first difference are more controlled. On the other hand, the most appropriate transformation of the indicators for the forecast of the exchange rate seems unclear, as it varies depending on the forecast time horizon that interests us more.

The VAR model performance with different selections reveals that for finding a suitable model for all variables, it may be necessary to include enough lags (usually 4), which seem to contain valuable information that leads to the improvement of the forecasts. In our exercise, this conclusion is again evident in the case of the projection of economic growth and inflation, especially if the model is estimated with the data in levels or in first differences. Meanwhile the interest rate and exchange rate forecast is less sensitive to the 1 to 4 time lags included in the model. The AIC, HQ and FPE information criteria, discussed above, suggested an abundant number of lags beyond the number tested here. However, the findings in this modest exercise seem to be more in line with Schwarz's criterion, which recommended significant lags for the VAR in levels and first differences, and 1 lag in estimations with annual changes.

Table 3 shows the forecast results of the model estimated by the Bayesian method, BVAR. Its shows a confirmation of the overall superiority of the model when variables are transformed into annual changes. Apart from re-emphasizing this form as best for forecasting economic growth and inflation, the Bayesian estimation reveals its usefulness for forecasting exchange rate as well (unlike the OLS method that, for the exchange rate, suggested a model with variables in first differences). Also, the Bayesian estimation confirms the importance of the information that is conveyed by an adequate number of lags (four lags if we refer to the loss function that minimizes forecast errors in case of having one model for all variables). However, the improvement of forecast ability by the Bayesian method, which significantly reduces the RMSEs for the exchange rate, does not appear so useful for all variables in the model, particularly for economic growth. Therefore, users are faced with the need of making trade-offs when selecting the estimation method, depending on the indicators that they are more interested in. However, these conclusions show the flexibility of the Bayesian method by keeping a considerate number of coefficients in the model, while at the same time reducing, in some ways, the concerns regarding the weakness of predictability of the VAR model, due to the significant number of estimated coefficients in relation to the number of observations.

Last but not least, a comparison of the recursive and rolling forecasts reveals that it could be better to extend the estimation sample period in our case. The results of a rather short rolling sample with 48 observations remain largely in line with the conclusions drawn from the recursive forecasts, with regard to variables transformation (yoy), recommendations on the number of lags, and

the advantages of each estimation method (please see Table 2A and 3A in the Appendix). However, the generally positive differences between RMSEs from rolling and recursive forecasts - although not that significant - point out the inability to improve forecast derived from the recursive strategy. Loss of information due to short samples and reduction of forecasting performance does not support the idea of structural breaks in the time series during our investigation period. This also implies that empirical analyses that use linear methods to estimate parameters can be reliable for the Albanian economy, at least for estimations that exclude the 1990s.

CONCLUDING REMARKS

This analysis discusses the nature of the VAR models for forecasting purposes based on the importance of stationarity, the use of information criteria, attention to dimensionality and structural breaks in the data.

Regarding the transformation of time series, estimations with data expressed as annual changes appeared to be the preferred form for forecasting our VAR variables, except the exchange rate. To improve the performance of the latter, it is worth using the Bayesian method for model evaluation; this may not be achieved without compromising the growth forecast performance. Having said that, the most accurate forecast of each indicator may require several adjustments of the model's evaluation and specification, since it is perhaps impossible to achieve this goal with one single VAR model.

Also, the results removes all doubts for the selection of many time series, particularly in unrestricted VAR models in level and first differences, and when it is used for forecasting purposes. Similarly, lost information due the short samples does not lead to the improvement of the models' predictive performance. This implies that the models' assessment for the Albanian economy with linear methods can be useful, at least for the evaluation samples that avoid the 1990s.

To further support our conclusion, the discussion in this article can be extended with measurements of forecast performance other than RMSE, such as the Measure of Change of Direction and the Diebold-Mariano statistics. The RMSE average shows the average performance of the model; their standard deviation can enrich the distribution of the model's performance throughout the repetitions of model trainings. The increase of the number of variables in VAR may shed light whether the VAR estimation method with OLS would maintain its performance in comparison to the Bayesian method.

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APPENDIX

Table 2A VAR Model Rolling Forecast: Average RMSE of Out-of-sample Period during 2013Q1:2018Q1

Variables, in %	Annual Growth			Annual Inflation			RER, yoy chg.			Policy rate		
Forecast horizon	1Q	1Y	2Y	1Q	1Y	2Y	1Q	1Y	2Y	1Q	1Y	2Y
Variable transformation												
Levels	3.0	3.2	3.3	0.8	1.0	1.0	1.4	2.2	2.8	0.1	0.4	0.8
YoY	1.7	1.8	1.8	0.4	0.5	0.7	1.0	2.4	4.1	0.2	0.4	0.7
QoQ	2.5	3.2	3.3	0.6	0.8	0.9	0.9	1.3	1.7	0.2	0.4	0.7
Model selection												
1 Lag	2.7	3.2	3.4	0.8	1.1	1.1	1.1	2.1	2.8	0.1	0.4	0.7
2 Lags	3.0	3.2	3.5	0.6	0.8	0.9	1.1	1.9	2.8	0.1	0.4	0.8
3 Lags	2.2	2.7	2.6	0.6	0.6	0.7	1.1	1.9	2.8	0.2	0.4	0.8
4 Lags	1.6	1.8	1.8	0.5	0.6	0.8	1.1	1.9	3.1	0.2	0.4	0.7
Model selection: VAR in levels												
Level: 1 lag	3.6	3.9	4.3	1.2	1.7	1.6	1.6	2.6	2.7	0.1	0.4	0.7
Level: 2 lags	3.8	3.6	3.9	0.8	1.1	1.1	1.5	2.4	2.8	0.1	0.3	0.8
Level: 3 lags	3.0	3.7	3.6	0.7	0.6	0.7	1.2	2.1	3.0	0.1	0.4	0.9
Level: 4 lags	1.6	1.6	1.5	0.6	0.7	0.7	1.2	1.8	2.8	0.2	0.3	0.6
Model selection: VAR in annual changes (YoY)												
YoY: 1 lag	1.3	1.5	1.6	0.4	0.5	0.7	1.0	2.4	4.0	0.2	0.3	0.6
YoY: 2 lags	2.0	2.1	2.0	0.4	0.6	0.7	0.9	2.3	4.0	0.2	0.4	0.8
YoY: 3 lags	1.7	1.9	1.8	0.5	0.5	0.7	0.9	2.2	3.7	0.1	0.3	0.7
YoY: 4 lags	1.7	1.9	1.8	0.4	0.5	0.8	1.1	2.7	4.8	0.2	0.4	0.9
Model selection: VAR in first difference (QoQ)												
QoQ: 1 lag	3.0	4.1	4.2	0.7	1.0	1.0	0.8	1.3	1.7	0.1	0.4	0.8
QoQ: 2 lags	3.3	4.1	4.5	0.7	0.8	0.9	0.8	1.2	1.6	0.1	0.4	0.7
QoQ: 3 lags	1.9	2.5	2.5	0.6	0.8	0.8	1.0	1.4	1.7	0.2	0.4	0.7
QoQ: 4 lags	1.6	2.0	2.0	0.5	0.7	0.8	0.9	1.3	1.7	0.2	0.4	0.7

Table 3A. BVAR Model Rolling Forecast Performance Average RMSE of Out-of-sample Period during 2013Q1:2018Q1

Variables, in %	Annual Growth			Annual Inflation			RER, yoy chg.			Policy rate		
Forecast horizon	1Q	1Y	2Y	1Q	1Y	2Y	1Q	1Y	2Y	1Q	1Y	2Y
Variable transformation												
Levels	7.3	6.5	5.0	1.3	1.5	1.3	1.5	1.8	1.8	0.1	0.3	0.4
YoY	1.5	2.0	2.6	0.4	0.6	0.6	0.7	1.0	1.2	0.2	0.3	0.7
QoQ	4.8	7.6	7.8	0.9	1.5	1.5	1.0	2.7	4.5	0.1	0.4	0.7
Model selection												
1 Lag	5.7	7.5	7.8	1.0	1.6	1.5	1.1	1.9	2.6	0.1	0.3	0.6
2 Lags	4.9	5.9	5.2	0.8	1.2	1.1	1.1	1.8	2.4	0.1	0.3	0.6
3 Lags	4.5	4.6	4.1	0.8	1.1	1.0	1.1	1.8	2.5	0.1	0.3	0.6
4 Lags	2.9	3.5	3.3	0.7	1.0	1.0	1.0	1.8	2.5	0.1	0.3	0.6
Model selection: VAR in levels												
Level: 1 lag	9.4	7.9	5.9	1.4	1.7	1.5	1.6	1.8	1.7	0.1	0.2	0.3
Level: 2 lags	8.0	7.3	5.4	1.4	1.7	1.4	1.6	1.8	1.8	0.1	0.3	0.4
Level: 3 lags	8.1	7.0	5.2	1.2	1.5	1.2	1.5	1.7	1.8	0.1	0.3	0.4
Level: 4 lags	3.7	3.9	3.3	1.0	1.2	1.0	1.3	1.6	1.8	0.1	0.3	0.5
Model selection: VAR in annual changes (YoY)												
YoY: 1 lag	1.5	2.0	2.6	0.3	0.6	0.6	0.6	0.9	1.2	0.2	0.3	0.7
YoY: 2 lags	1.5	2.0	2.6	0.3	0.6	0.6	0.7	0.9	1.2	0.2	0.3	0.7
YoY: 3 lags	1.5	2.0	2.6	0.4	0.6	0.6	0.7	1.0	1.2	0.2	0.3	0.7
YoY: 4 lags	1.5	1.9	2.6	0.4	0.6	0.6	0.7	1.0	1.3	0.2	0.4	0.8
Model selection: VAR in first difference (QoQ)												
QoQ: 1 lag	6.4	12.6	15.0	1.3	2.5	2.4	1.1	2.9	4.9	0.1	0.3	0.7
QoQ: 2 lags	5.3	8.2	7.6	0.8	1.2	1.2	1.0	2.6	4.4	0.1	0.4	0.7
QoQ: 3 lags	4.0	4.9	4.4	0.7	1.2	1.2	1.0	2.7	4.4	0.1	0.4	0.7
QoQ: 4 lags	3.7	4.5	4.0	0.7	1.2	1.2	1.0	2.6	4.4	0.1	0.4	0.7